OCR
Oxford Cambridge and RSA

Tuesday 21 June 2016 – Morning

A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4723/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions

1 Find the equation of the tangent to the curve

$$y = 3x^2(x+2)^6$$

at the point $(-1, 3)$, giving your answer in the form $y = mx + c$. **[5]**

2 Find

(i) $\int \left(2 - \frac{1}{x}\right)^2 \mathrm{d}x$,

(ii) $\int (4x+1)^{\frac{1}{3}} \mathrm{d}x$.

[5]

3 The mass of a substance is decreasing exponentially. Its mass is m grams at time t years. The following table shows certain values of t and m.

t	0	5	10	25
m	200	160		

(i) Find the values missing from the table. **[2]**

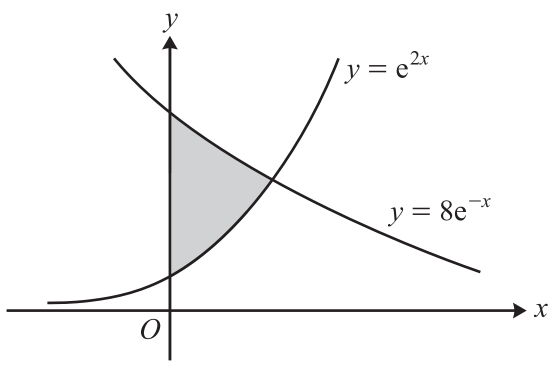
(ii) Determine the value of t, correct to the nearest integer, for which the mass is 50 grams. **[4]**

4 It is given that A and B are angles such that

$$\sec^2 A - \tan A = 13 \quad \text{and} \quad \sin B \sec^2 B = 27 \cos B \operatorname{cosec}^2 B.$$

Find the possible exact values of $\tan(A - B)$. **[8]**

5



The diagram shows the curves $y = \mathrm{e}^{2x}$ and $y = 8\mathrm{e}^{-x}$. The shaded region is bounded by the curves and the y-axis. Without using a calculator,

(i) solve an appropriate equation to show that the curves intersect at a point for which $x = \ln 2$, **[2]**

(ii) find the area of the shaded region, giving your answer in simplified form. **[5]**

6 The curves C_1 and C_2 have equations

$$y = \ln(4x-7)+18 \quad \text{and} \quad y = a(x^2+b)^{\frac{1}{2}}$$

respectively, where a and b are positive constants. The point P lies on both curves and has x-coordinate 2. It is given that the gradient of C_1 at P is equal to the gradient of C_2 at P. Find the values of a and b. **[8]**

7 **(i)** By sketching the curves $y = x(2x+5)$ and $y = \cos^{-1}x$ (where y is in radians) in a single diagram, show that the equation $x(2x+5) = \cos^{-1}x$ has exactly one real root. **[3]**

(ii) Use the iterative formula

$$x_{n+1} = \frac{\cos^{-1}x_n}{2x_n+5} \quad \text{with} \quad x_1 = 0.25$$

to find the root correct to 3 significant figures. Show the result of each iteration correct to at least 4 significant figures. **[4]**

(iii) Two new curves are obtained by transforming each of the curves $y = x(2x+5)$ and $y = \cos^{-1}x$ by the pair of transformations:

reflection in the x-axis followed by reflection in the y-axis.

State an equation of each of the new curves and determine the coordinates of their point of intersection, giving each coordinate correct to 3 significant figures. **[4]**

8 The functions f and g are defined for all real values of x by

$$\text{f}(x) = |2x+a|+3a \quad \text{and} \quad \text{g}(x) = 5x-4a,$$

where a is a positive constant.

(i) State the range of f and the range of g. **[2]**

(ii) State why f has no inverse, and find an expression for $\text{g}^{-1}(x)$. **[3]**

(iii) Solve for x the equation $\text{gf}(x) = 31a$. **[5]**

9 **(i)** Show that $\sin 2\theta(\tan\theta + \cot\theta) \equiv 2$. **[4]**

(ii) Hence

(a) find the exact value of $\tan\frac{1}{12}\pi + \tan\frac{1}{8}\pi + \cot\frac{1}{12}\pi + \cot\frac{1}{8}\pi$, **[3]**

(b) solve the equation $\sin 4\theta(\tan\theta + \cot\theta) = 1$ for $0 < \theta < \frac{1}{2}\pi$, **[3]**

(c) express $(1-\cos 2\theta)^2\left(\tan\frac{1}{2}\theta + \cot\frac{1}{2}\theta\right)^3$ in terms of $\sin\theta$. **[2]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance
1			Differentiate to produce form $k_1x(x+2)^m + k_2x^2(x+2)^n$	*M1	For positive integers k_1, k_2, m, n; allow M1 if slip to, for example, $(x+3)$ in both brackets
			Obtain $6x(x+2)^6 + 18x^2(x+2)^5$	A1	Or unsimplified equiv
			Substitute $x=-1$ to obtain value 12	A1	From correct work only
			Attempt equation of tangent (not normal) through point $(-1, 3)$	M1	Dep *M; using non-zero numerical value of gradient; condone slip in use of coordinates
			Obtain $y = 12x + 15$	A1	Answer required in $y = mx + c$ form
				[5]	
2	i		Expand to produce form $k_1 + \frac{k_2}{x} + \frac{k_3}{x^2}$	M1	For non-zero constants k_1, k_2, k_3; allow if middle term appears as two, so far, unsimplified terms
			Obtain $4x - 4\ln x - \frac{1}{x}$ or $4x - 4\ln x - x^{-1}$	A1	Condoning absence of modulus signs but A0 if expression involves $\lvert\ln x\rvert$ or $\lvert 4\ln x\rvert$
	ii		Integrate to obtain form $k(4x+1)^{\frac{4}{3}}$	M1	Any non-zero constant k
			Obtain $\frac{3}{16}(4x+1)^{\frac{4}{3}}$	A1	With coefficient simplified
			Include $\ldots + c$ or $\ldots + k$ at least once anywhere in answer to question 2	B1	Even if associated with incorrect integral
				[5]	

Question			Answer	Marks	Guidance	
3	i		Obtain 128 for value corresponding to 10 Obtain 65.5 for value corresponding to 25	B1 B1 **[2]**	Allow any value rounding to 128 Allow any value rounding to 65 or 66; whether obtained using powers of 0.8 or by use of formula	
	ii		Attempt to find formula for m of form $200\mathrm{e}^{kt}$ or $200 \times r^{\lambda t}$	M1	Whether attempted in part (i) or (ii)	If formula attempted in part (i), marks earned must be recorded in part (ii)
			Obtain $200\mathrm{e}^{(0.2\ln 0.8)t}$ or $200\mathrm{e}^{-0.0446t}$ or $200 \times 0.8^{0.2t}$ or 200×0.956^{t}	A1	Or equiv	
			Show correct process for solving equation of form $200\mathrm{e}^{kt} = 50$ or $200r^{\lambda t} = 50$	M1		
			Obtain 31	A1 **[4]**	Or greater accuracy rounding to 31; ignore any units given; second M1 is implied by correct answer	Special case: no formula anywhere and answer 31 (or greater accuracy) given, award B2 (i.e. 2/4 for part (ii))
4			Use identity $\sec^2 A = 1 + \tan^2 A$	B1		
			Attempt solution of three-term quadratic equation to obtain two values of $\tan A$	M1	Implied if correct values obtained; allow M1 for incorrect factorisation provided expansion would give correct first and third terms; allow M1 for incorrect use of formula if only one error present	
			Obtain $\tan A = -3$ and $\tan A = 4$	A1	And no others; implied by $A = \tan^{-1} -3$ and $\tan^{-1} 4$;	$A = -3, 4$ is A0 here unless subsequent work shows values used correctly
			Use correct identities to produce equation in $\tan B$ only	M1	Equation might be $t^3 = 27$ …	… or $t^5 + t^3 - 27t^2 - 27 = 0$
			State $\tan B = 3$	A1	And no others	

Question			Answer	Marks	Guidance	
			Substitute at least one pair of non-zero numerical values into $\frac{\tan A - \tan B}{1 + \tan A \tan B}$	M1	Must be the correct identity	
			Obtain one of $\frac{1}{13}$ and $\frac{3}{4}$ or exact equiv	A1		
			Obtain the other exact value or equiv	A1 **[8]**	And no others	
5	i		Either State $e^{2x} = 8e^{-x}$ and so $e^{3x} = 8$	B1		Verifying by substitution of $\ln 2$ in each equation earns B0B0
			Obtain $e^x = 2$ and hence $x = \ln 2$	B1	AG; necessary detail needed	
			Or 1 State $e^{2x} = 8e^{-x}$ and so $e^{3x} = 8$	B1		
			State $3x = \ln 8$, $x = \ln 8^{\frac{1}{3}}$ and hence $x = \ln 2$	B1	AG; necessary detail needed	Going immediately from $x = \frac{1}{3}\ln 8$ to $x = \ln 2$ does not earn the second B1
			Or 2 State $e^{2x} = 8e^{-x}$ and $2x = \ln 8 - x$	B1		
			State $3x = \ln 8$, $x = \ln 8^{\frac{1}{3}}$ and hence $x = \ln 2$	B1 **[2]**	AG; necessary detail needed	Going immediately from $x = \frac{1}{3}\ln 8$ to $x = \ln 2$ does not earn the second B1
	ii		Integrate to obtain $k_1 e^{-x}$ and $k_2 e^{2x}$	M1	Any non-zero constants k_1 and k_2	
			Obtain correct $-8e^{-x} - \frac{1}{2}e^{2x}$ or, if done separately, $-8e^{-x}$ and $\frac{1}{2}e^{2x}$	A1		
			Apply limits 0 and $\ln 2$ correctly to their integral(s)	M1	Condone one sign slip; earned by sight of $-8e^{-\ln 2} - \frac{1}{2}e^{2\ln 2} + 8 + \frac{1}{2}$ (or equivs if integrals treated separately)	M1 also implied by sight only of $-4 - 2 + 8 + \frac{1}{2}$ (or equivs …)
			Obtain at least $-4 - 2 + 8 + \frac{1}{2}$ (or equivs)	*A1		
			Obtain $\frac{5}{2}$ or equiv	A1 **[5]**	Final A1 dependent on *A1	

Question			Answer	Marks	Guidance	
6			State, at some stage, $a(4+b)^{\frac{1}{2}}=18$	B1		
			Obtain derivative $\frac{4}{4x-7}$ for C_1	B1		
			Obtain derivative $kx(x^2+b)^{-\frac{1}{2}}$ for C_2	M1	Any non-zero constant k	
			Obtain correct $ax(x^2+b)^{-\frac{1}{2}}$	A1		
			Equate derivatives with $x=2$	M1		
			Attempt values of a and b from two equations involving a and $(4+b)^{\frac{1}{2}}$	M1	Using correct process Correct equations are $a(4+b)^{\frac{1}{2}}=18$ and $2a(4+b)^{-\frac{1}{2}}=4$	
			Obtain $a=6$	A1		
			Obtain $b=5$	A1		
				[8]		
7	i		Draw more or less correct sketch of $y=\cos^{-1}x$ existing in first and second quadrants	*B1	Ignore any curve outside $0\le y\le\pi$; condone no or wrong intercepts on axes	
			Draw U-shaped parabola passing through origin and showing minimum point	*B1	Curve must exist in first and third quadrants	
			Indicate one intersection in first quadrant by blob or reference in words or …	B1	Dep *B *B	
				[3]		

Question			Answer	Marks	Guidance	
	ii		Obtain correct first iterate showing at least 4 s.f. rounded or truncated	B1		
			Show iterative process to produce at least three iterates in all showing at least 3 s.f.	M1	Implied by incorrect values apparently converging	0.25
			Obtain at least four correct iterates in all showing at least 4 s.f.	A1	Allowing recovery after error	0.23965… 0.24250…
			Conclude with value 0.242	A1	Answer to be clearly indicated by underlining final value in sequence or by separate statement; answer required to precisely 3 s.f.; allow final A1 even if iterates have been shown to only 3 s.f.; answer only earns 0/4	0.24172… 0.24193…
				[4]		
	iii		State $y = -\cos^{-1}(-x)$ or $y = \cos^{-1} x - \pi$	B1		
			State $y = x(-2x+5)$ or equiv	B1	Allow $y = --x(2(-x)+5)$ or similar; condone missing $y =$ in each case	
			State -0.242 for x-coordinate	B1 FT	Following their answer to **(ii)**; allow greater accuracy here	
			State -1.33 for y-coordinate	B1	Allow value rounding to -1.33	
				[4]		
8	i		State range of f is $f(x) \geq 3a$ or $y \geq 3a$	B1	Allow $f \geq 3a$ or equiv expression in words but $3a$ to be included	
			State range of g is all real numbers or equiv such as $y \in R$ (real numbers)	B1		
				[2]		

Question			Answer	Marks	Guidance
	ii		State function is not 1 – 1 or different x-values give same y-value or equiv	B1	no credit for 'no inverse due to modulus' nor for 'cannot be reflected across $y=x$'
			Obtain form $k(y+4a)$ or $k(x+4a)$	M1	for non-zero constant k
			Obtain $\frac{1}{5}(x+4a)$ or $\frac{1}{5}x+\frac{4}{5}a$	A1 **[3]**	Must finally be in terms of x
	iii		Either Attempt composition of functions the right way round	M1	Earned for 5(what they think f(x) is) – $4a$
			Obtain $5\lvert 2x+a\rvert+11a=31a$ or equiv	A1	
			Or Apply their g^{-1} to $31a$	M1	
			Obtain $\lvert 2x+a\rvert+3a=7a$ or equiv	A1	
			Either Solve $2x+a=4a$ and obtain $\frac{3}{2}a$	B1 FT	Following their $\lvert 2x+a\rvert=ka$
			Solve linear equation in which signs of (their) $2x$ and (their) $4a$ are different	M1	Condone other sign slips
			Obtain $-\frac{5}{2}a$	A1	And no others; obtaining $-\frac{5}{2}a$ and then concluding $\frac{5}{2}a$ is A0
			Or Square both sides and obtain $4x^2+4ax-15a^2=0$	B1 FT	Following their $\lvert 2x+a\rvert=ka$
			Solve 3-term quadratic equation to obtain two values	M1	Allow M1 if factorisation wrong but expansion gives correct first and third terms; allow M1 if incorrect use of formula involves only one error
			Obtain $-\frac{5}{2}a$, $\frac{3}{2}a$	A1 **[5]**	And no others; continuing from two correct answers to conclude $\frac{5}{2}a$, $\frac{3}{2}a$ is A0

Question			Answer	Marks	Guidance	
9	i		Use $\sin 2\theta = 2\sin\theta\cos\theta$	B1		Note that going directly from $2\sin^2\theta + 2\cos^2\theta$ to 2 is M0 but $2(\sin^2\theta + \cos^2\theta)$ to 2 is M1A1
			State $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$ or $\tan\theta + \frac{1}{\tan\theta}$	B1	Perhaps as part of expression	
			Simplify using correct identities	M1		
			Obtain 2 correctly	A1	AG; necessary detail needed	
				[4]		
	ii	a	Obtain expression involving at least one of $\sin\frac{1}{6}\pi$ and $\sin\frac{1}{4}\pi$	M1		
			Obtain $\frac{2}{\sin\frac{1}{6}\pi} + \frac{2}{\sin\frac{1}{4}\pi}$	A1	Or equiv involving cosecant	
			Obtain $4 + 2\sqrt{2}$ or exact equiv	A1	Answer only is 0/3	
				[3]		
		b	Use $\sin 4\theta = 2\sin 2\theta\cos 2\theta$	B1		
			Obtain $\cos 2\theta = \frac{1}{4}$ or $\cos^2\theta = \frac{5}{8}$ or $\sin^2\theta = \frac{3}{8}$	B1		
			Obtain 0.659 or 0.66	B1	Or greater accuracy; and no others between 0 and $\frac{1}{2}\pi$; allow 0.21π but not 0.659π ; answer only earns 0/3	
				[3]		
		c	Express in form $k_1\sin^4\theta \times \frac{k_2}{\sin^3\theta}$	M1		
			Obtain $4\sin^4\theta \times \frac{8}{\sin^3\theta}$ and hence $32\sin\theta$	A1	A0 if $(-2\sin^2\theta)^2$ involved in simplification	
				[2]		